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233. Proposed by PROF. R. D. CARMICHAEL, Anniston, Ala.

Evaluate (a)  $\int_0^{\frac{1}{2}\pi} \frac{\sin nx}{\sin x} dx$ , and (b)  $\int_0^{\frac{1}{2}\pi} \frac{\sin^2 nx}{\sin x} dx$ , where  $n$  is a positive integer.

Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va., and J. SCHEFFER, A. M., Hagerstown, Md.

$$(a) \frac{\sin nx}{\sin x} = 2[\cos x + \cos 3x + \cos 5x + \dots + \cos (n-1)x], \quad n \text{ even,}$$

$$= 1 + 2[\cos 2x + \cos 4x + \cos 6x + \dots + \cos (n-1)x], \quad n \text{ odd.}$$

$$\begin{aligned} \int_0^{\frac{1}{2}\pi} \frac{\sin nx}{\sin x} dx &= 2 \left( \sin x + \frac{1}{3}\sin 3x + \frac{1}{5}\sin 5x + \dots + \frac{1}{n-1}\sin (n-1)x \right)_0^{\frac{1}{2}\pi} \\ &= 2 \left( 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \dots \pm \frac{1}{n-1} \right), \quad n \text{ even} = \frac{1}{2}\pi \text{ if } n = \infty. \end{aligned}$$

$$\begin{aligned} \int_0^{\frac{1}{2}\pi} \frac{\sin nx}{\sin x} dx &= \left( x + \sin 2x + \frac{1}{2}\sin 4x + \frac{1}{3}\sin 6x + \dots - \frac{2}{n-1}\sin (n-1)x \right)_0^{\frac{1}{2}\pi} \\ &= \frac{1}{2}\pi \text{ when } n \text{ is odd.} \end{aligned}$$

$$(b) \frac{\sin^2 nx}{\sin x} = \sin x + \sin 3x + \sin 5x + \dots + \sin (2n-1)x.$$

$$\begin{aligned} \int_0^{\frac{1}{2}\pi} \frac{\sin^2 nx}{\sin x} dx &= - \left( \cos x + \frac{1}{3}\cos 3x + \frac{1}{5}\cos 5x + \dots + \frac{1}{2n-1}\cos (2n-1)x \right)_0^{\frac{1}{2}\pi} \\ &= 1 + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \dots + \frac{1}{2n-1}. \end{aligned}$$

Also solved by Prof. F. Anderegg.

## DIOPHANTINE ANALYSIS.

139. Proposed by PROF. R. D. CARMICHAEL, Anniston, Ala.

$2^{n-1}(2^n-1)$  is a multiply perfect number of multiplicity 2 when  $2^n-1$  is prime. Prove that there are no other multiply perfect numbers containing only 2 distinct primes.

Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

Let  $2^n-1=b$ .

$$\text{Then } \frac{2^n-1}{2^{n-1}(2^n-1)} \cdot \frac{b^2-1}{b(b-1)} = \frac{2^n-1}{2^{n-1}} \cdot \frac{b+1}{b} = \frac{2^n-1}{2^{n-1}} \cdot \frac{2^n}{2^n-1} = 2.$$

$\therefore 2^{n-1}(2^n-1)$  is a multiply perfect number of multiplicity 2. The second part of the problem is demonstrated in problem 137, Vol. XIII, Nos. 8-9.